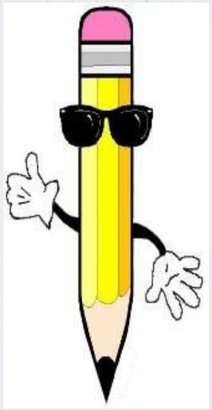
Coordinate Plane

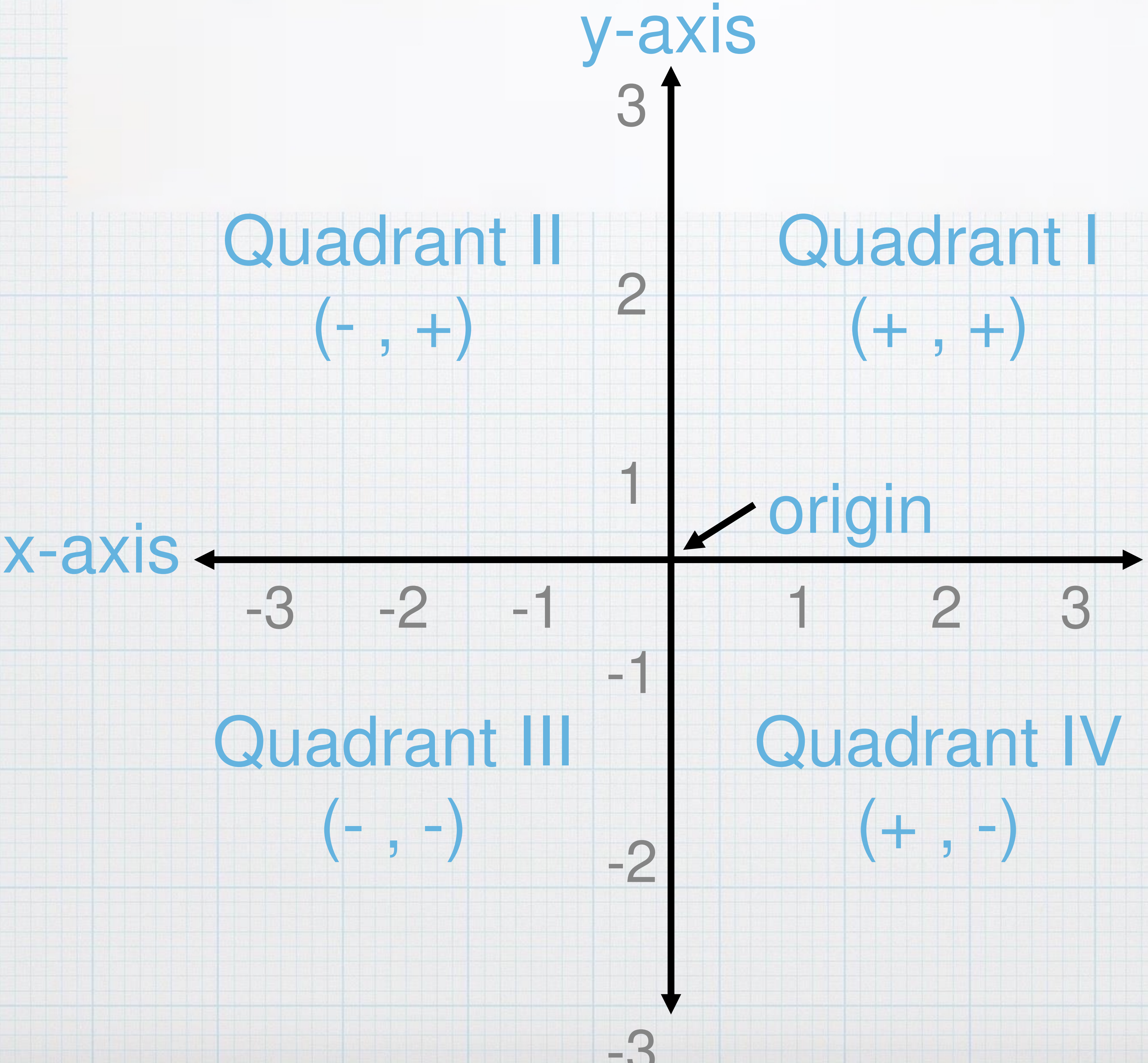
Essential Questions: What are the different pieces of a coordinate plane? How do you plot points on a coordinate plane?

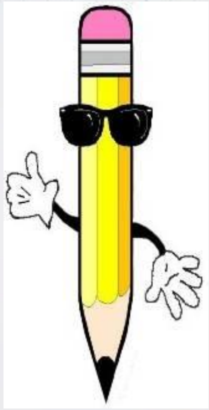
Vocabulary

* Coordinate Plane: is formed by two lines that intersect at a right angle
* x-axis: the **horizontal** axis on a coordinate plane
* y-axis: the **vertical** axis on a coordinate plane
* Origin: where the x-axis and y-axis **intersect** at (0 , 0)
* Ordered Pair: a **point** you plot on a coordinate plane in the form (x , y)
* Scatter Plot: a type of graph that is used to see trends in data and to make predictions by plotting points but not connecting them with a line

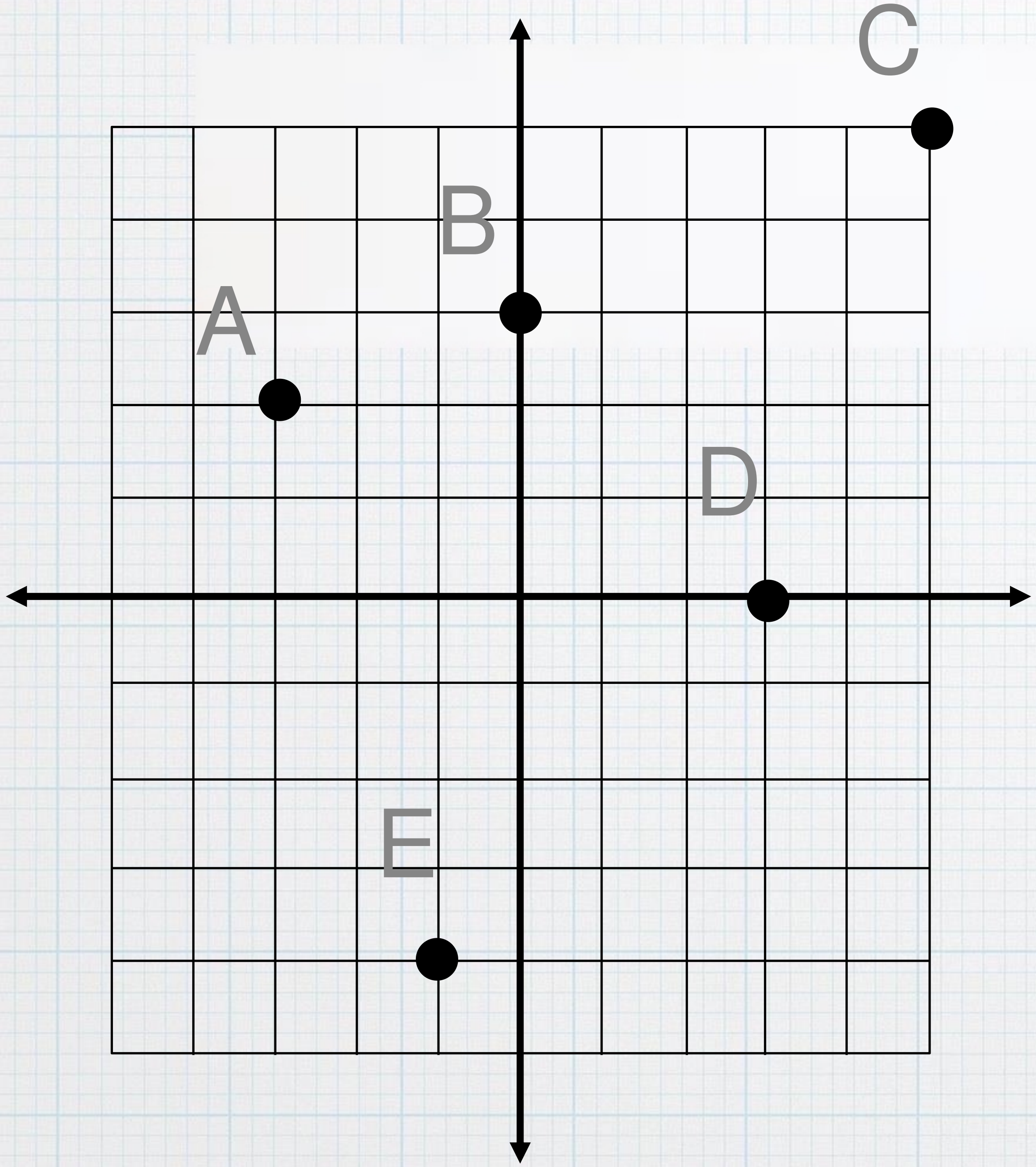


The Coordinate Plane





xample 1: Give the ordered pair for the following points.



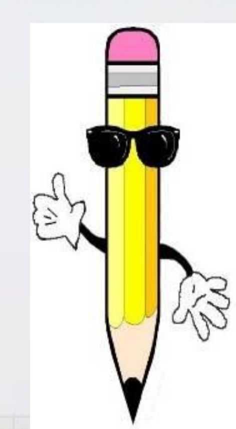
Point A (-3 , 2)

Point B (0 , 3)

Point C (5 , 5)

Point D (3 , 0)

Point E (-1 , -4)



Example 2: Without graphing, name the quadrant or axis location of each point.

(2 , 1) Quadrant I

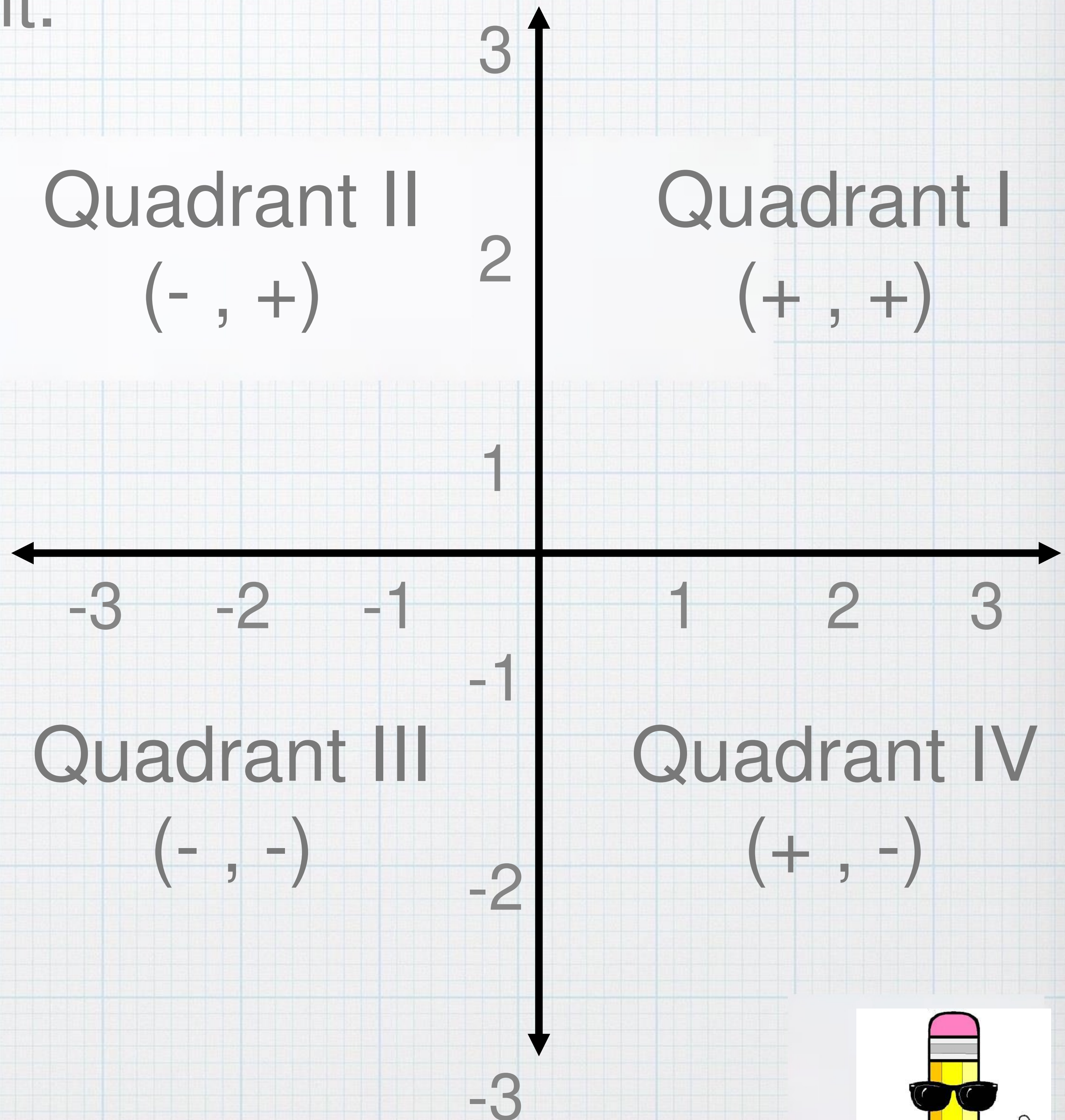
(-3 , 0) x-axis

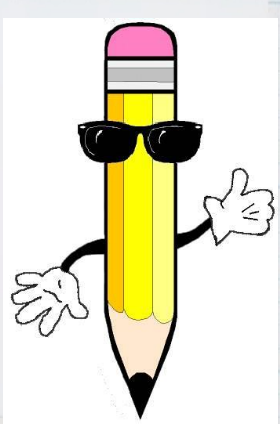
(5 , -3) Quadrant IV

(-2 , -2) Quadrant III

(0 , 4) y-axis

(-2 , 3) Quadrant II





Example 3: Plot and label the points from example 2.

A (2 , 1)

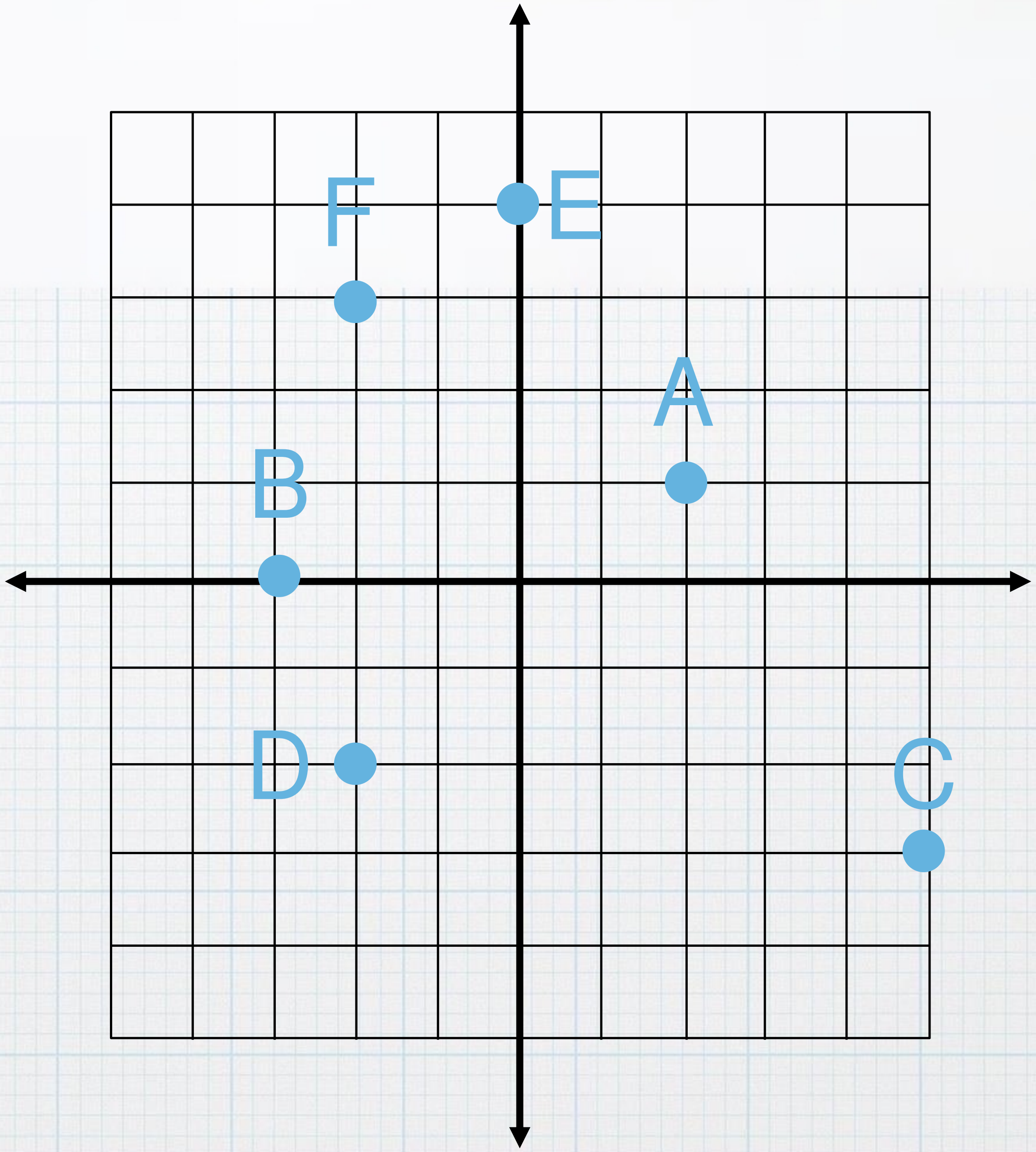
B (-3 , 0)

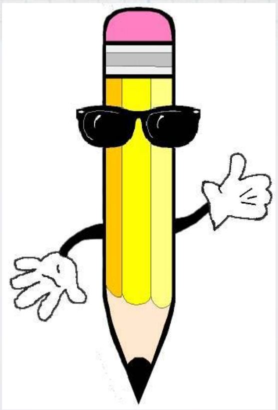
C (5 , -3)

D (-2 , -2)

E (0 , 4)

F (-2 , 3)

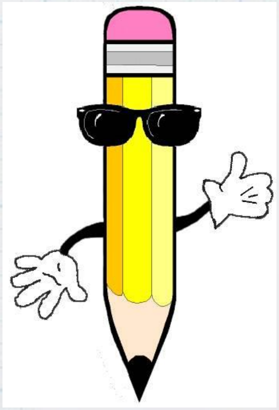




Summary

<u>Essential Questions:</u> What are the different pieces of a coordinate plane? How do you plot points on a coordinate plane?

Take 1 minute to write 2 sentences answering the essential questions.

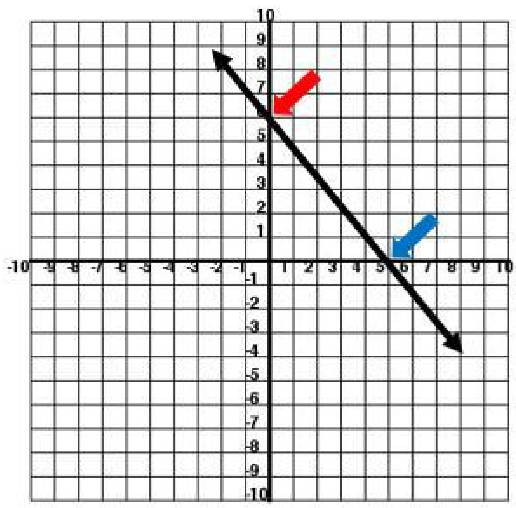


Linear Functions

X and Y Intercepts of Linear Functions

You will learn how to identify and/or solve for the x-intercept and the y-intercept of a function.

Example A



To find the x-intercept from a graph, determine the point at which the graph intersects the x-axis.

The x-intercept is (5, 0).

To find the y-intercept from a graph, determine the point at which the graph intersects the y-axis.

The y-intercept is (0, 6).

Example B

To find the x and y intercepts from a table, look for the number zero.

x	y
-4	0
-3	1
-2	2
-1	3
0	4
1	5

(-4, 0) is the x-intercept because the y value is 0

(0, 4) is the y-intercept because the x value is 0

x	y
-5	8
0	7
5	6
10	5
15	4
20	3
25	2
30	1
35	0

(0, 7) is the y-intercept because the x value is 0

The x-intercept is not visible in the graph. You must continue the pattern to find the other number zero.

(35, 0) is the x-intercept because the y value is 0

Example C

$-2x + 3y = 12$

To find the x-intercept from an equation, substitute 0 for y and solve for x.

$-2x + 3(0) = 12$

$-2x + 0 = 12$

$$\frac{-2x}{-2} = \frac{12}{-2}$$

$x = -6$

The x-intercept is (-6, 0)

$-2x + 3y = 12$

To find the y-intercept from an equation, substitute 0 for x and solve for y.

$-2(0) + 3y = 12$

$0 + 3y = 12$

$$\frac{3y}{3} = \frac{12}{3}$$

$y = 4$

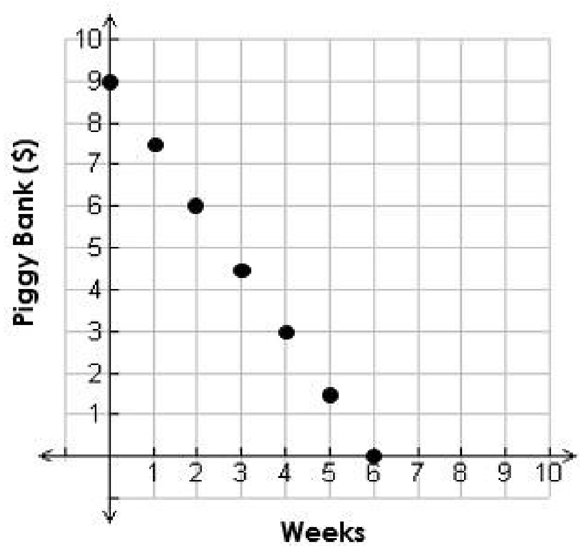
The y-intercept is (0, 4)

Example D

Manuel started with \$9 in his piggy bank. Each week he takes out \$1.50 for a snack. The amount of money in his piggy bank after x weeks is represented by the function $f(x) = 9 - 1.5x$.

The y-intercept is (0, 9) because at 0 weeks Manuel has 9 dollars in his piggy bank.

The x-intercept is (6, 0) because after 6 weeks, Manuel has 0 dollars left in his piggy bank.



Vocabulary

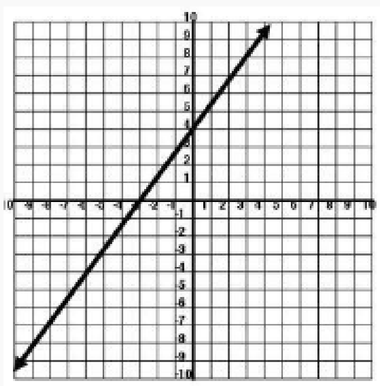
x-Intercept = the point at which a line crosses the x axis and the y value is 0. y-Intercept = the point at which a line crosses the y axis and the x value is 0.

Independent Practice

Find the x and y intercepts. Write your answers as order pairs.

TABLE 1.1:

1.



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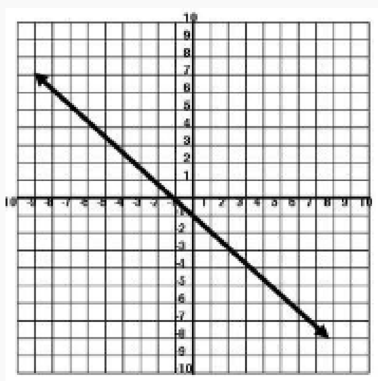

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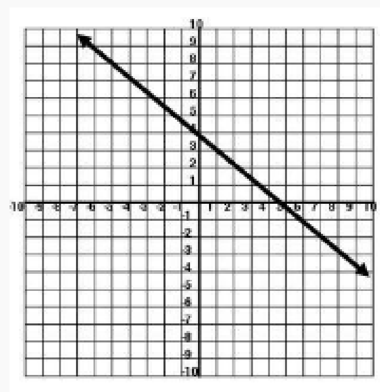


TABLE 1.2:

4.

x	y
-3	-2
-2	0
-1	2
0	4
1	6

5.

x	0	3	6	9	12
y	-4	-3	-2	-1	0

6.

x	y
0	5
2	6
4	7
6	8
8	9

7.

x	-4	-3	-2	-1
y	0	6	12	18

Find the x and y intercepts from the following equations.

TABLE 1.3:

8. $3x - y = 3$	9. $3y - 2x = 6$	10. $2x = 4y - 8$

Find the x and y intercepts from the following equations and graph the line.

TABLE 1.4:

11. $8x - 3y = 24$	12. $5x - 4y = 20$	13. $7x + 3y = 21$	14. $-7x + 2y = 14$

Using intercepts in real world situations.

15.

Rosa has $30 to spend on refreshments for herself and her friends at the carnival. The equation $3x + 2y = 30$ describes the number of small popcorns, x, and small drinks, y, she can buy. Graph this function, find its intercepts and describe the meaning of each intercept.

x-intercept __________

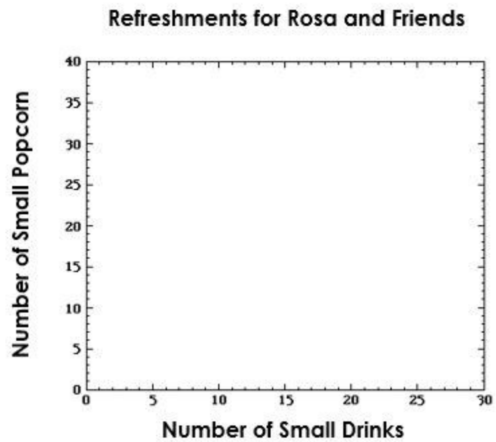
Meaning of x-intercept

____
____

y-intercept __________

Meaning of y-intercept

____
____



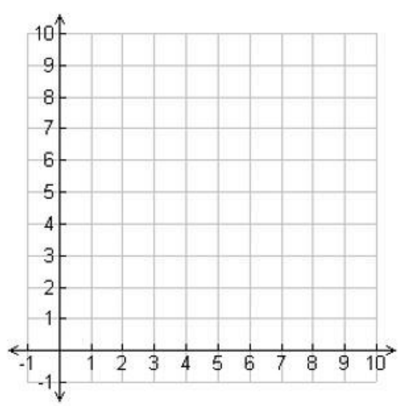
16.

At the local grocery store, strawberries cost $3.00 per pound and bananas cost $1.00 per pound.

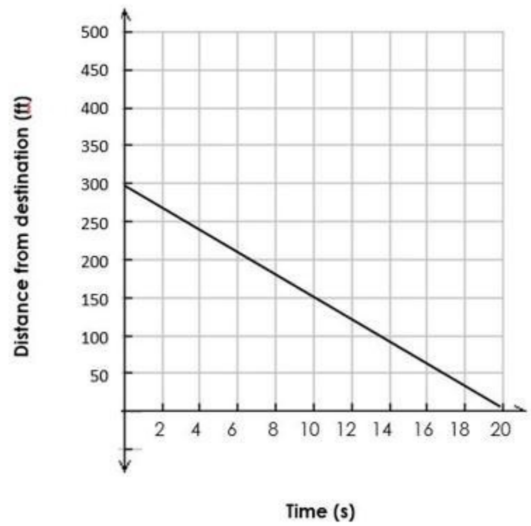
a. If I have $10 to spend on strawberries and bananas, draw a graph to show what combinations of each I can buy and spend exactly $10.

b. Plot the point representing 3 pounds of strawberries and 2 pounds of bananas. Will that cost more or less than $10?

c. Do the same for the point representing 1 pound of strawberries and 5 pounds of bananas.



The graph shows the distance of an elevator at the First National Bank, from its destination as a function of time. Use the graph to answer the following questions.



17. What is the x-intercept?

    a. 20 b. 0 c. 500 d. 300

18. What does the x-intercept represent?

    a. The time it takes to reach the first floor.
    b. The total distance the elevator travels.
    c. The distance that the elevator has traveled at any time.
    d. The number of seconds that have passed for any given distance.

19. What is the y-intercept?

    a. 300 b. 0 c. 20 d. 500

20. What does the y-intercept represent?

    a. The total distance the elevator travels.
    b. The number of seconds that have passed for any given distance.
    c. The time it takes to reach the first floor.
    d. The distance that the elevator has traveled at any time.

21.

Thomas is reading a 500 page book. He reads 5 pages every 8 minutes. The number of pages Thomas has left to read after x minutes is presented by the function $f(x) = 500 - \frac{5}{8}x$. Graph this function using the intercepts.

a. Meaning of y-intercept.

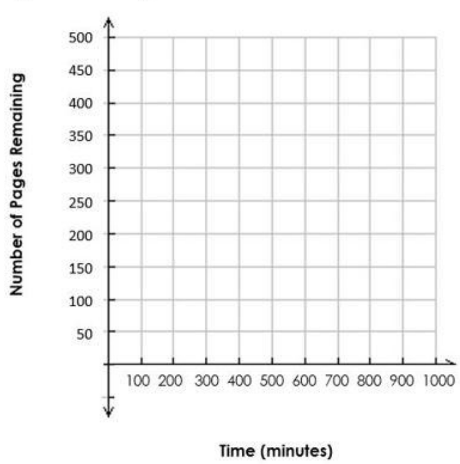
____

b. Meaning of x-intercept.

____

c. After how many minutes does Thomas have 350 pages left?

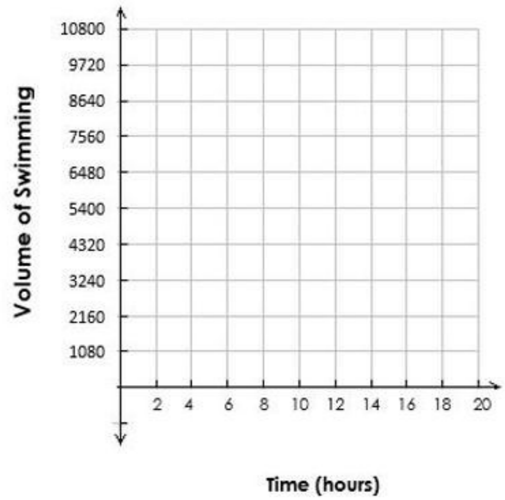
____



22.

Mathew is draining his swimming pool. The pool has a capacity of 10,800 gallons and the water level drops at a rate of 540 gal/hr.

How long will it take to empty the swimming pool? __________

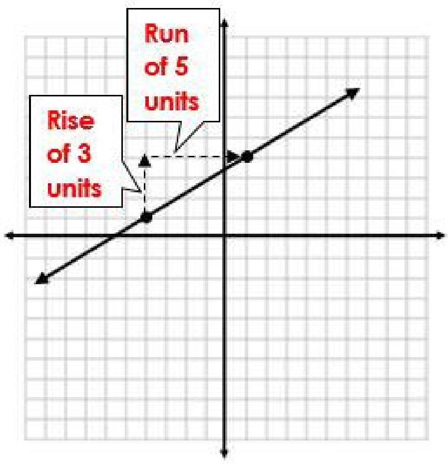


- intercepts
- x-intercept
- y-intercept
- x-axis
- y-axis
- zero

Slope of Linear Functions

You will be able to identify and calculate the slope of a real world situation given a table, a set of ordered pairs, an equation, a graph or context.

Example A



To find the **slope/m/rate of change** of a given line, find 2 points on the graph.

Calculate the $\dfrac{\text{change in y (RISE)}}{\text{change in x (RUN)}}$

Begin at the left most point. Count vertically to find the change in y-values (rise). Then count horizontally to the second point to find the change in x-values (run).

slope/m/rate of change = $\frac{3}{5}$

Example B

(-4, 2) and (-6, 10).

To find the **slope/m/rate of change** from given points, use the slope formula.

Formula for Slope of a Line – $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Label the ordered pairs. $\overset{x_1\ \ y_1}{(-4, 2)}$ $\overset{x_2\ \ y_2}{(-6, 10)}$

Substitute the values into the formula and simplify.

$$m = \frac{10 - 2}{-6 - -4} = \frac{8}{-2} = -4$$

slope/m/rate of change = -4

Example C

x	y
-3	-5
1	3
3	7
6	13

To find the slope from a table, pick any two points and use the Slope of a Line Formula. Follow the same steps as shown above.

Vocabulary

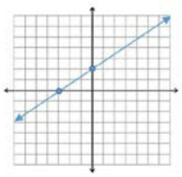
Slope = the measure of the steepness of a line.

Independent Practice

Find the slope/m/rate of change of the given lines.

TABLE 1.5:

1.



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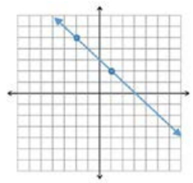

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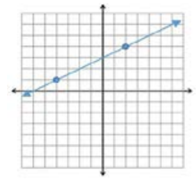

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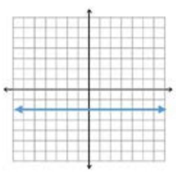

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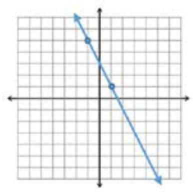

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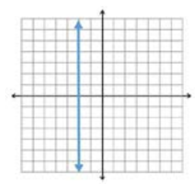

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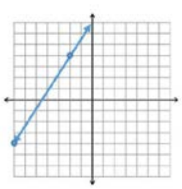

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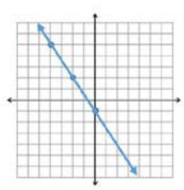

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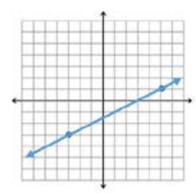

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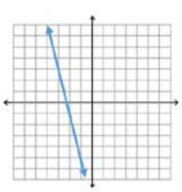
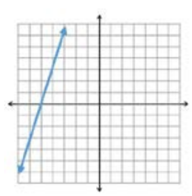


11.



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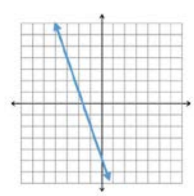


Find the slope/m/rate of change of a line containing the given points.

TABLE 1.6:

1. (2, 5) and (3, 7)	2. (3, 1) and (-1, 5)
3. (-1, 3) and (2, 7)	4. (0, 2) and (1, 5)
5. (2, -1) and (-3, 6)	6. (12, 6) and (-8, 6)
7. (-5, -2) and (0, 8)	8. (9, 7) and (9, -4)
9. (0, 2) and (-6, -2)	10. (3, 7) and (3, 10)

Each table shows a linear relationship. Find the slope.

TABLE 1.7:

11.

x	y
-4	-10
-2	-4
0	2
2	8

12.

x	y
10	-5
12	-5
14	-5
16	-5

13.

x	y
0	-2
-5	-6
-10	-10
-15	-14

14.

x	y
-3	-5
-1	1
2	10
5	19

15.

x	y
0	-2
0	-6
0	-10
0	-14

16.

x	y
0	-1
5	-5
10	-9
15	-13

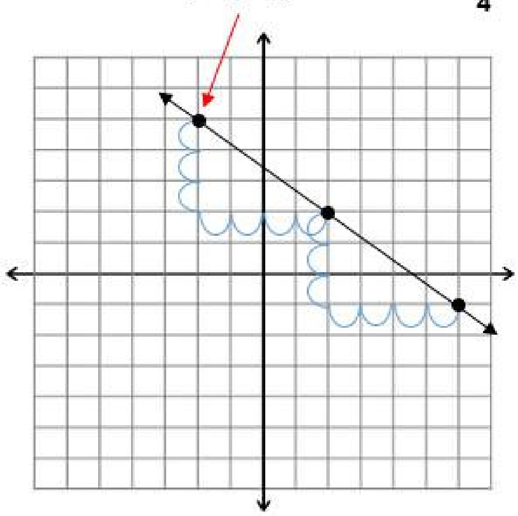
- slope
- rate of change

Graphing Linear Functions

You will be able to graph the line of any given equation on a coordinate plane and identify its key features.

Example A

Point (-2, 5) $m = -\frac{3}{4}$

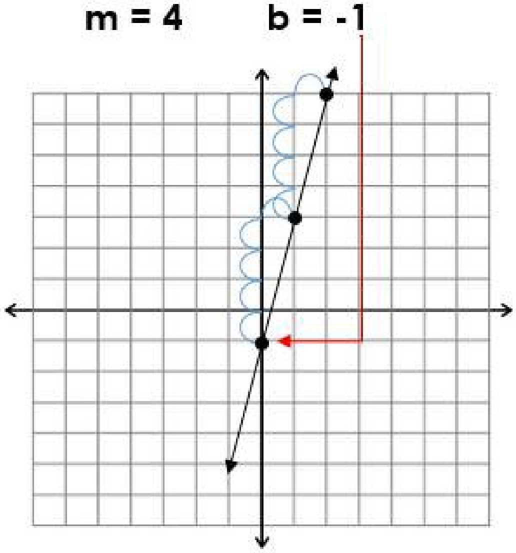


To graph a line when given a point and the slope(m), begin by first plotting the point.

Then use the slope(m) to find the next point. Since the slope(m) is negative, count down 3 units.
Since the denominator is 4, count right 4 units. Continue the pattern and graph the line.

Example B

$m = 4$ $b = -1$

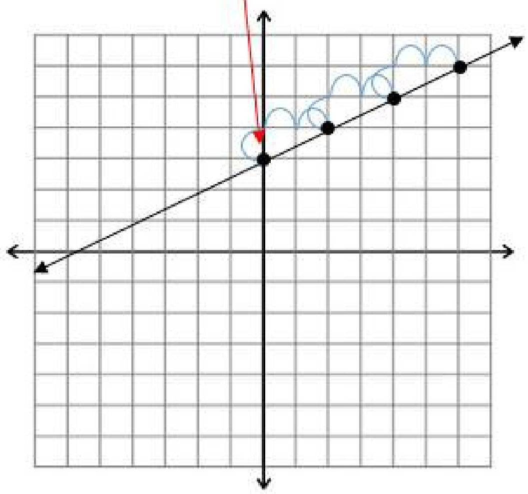


To graph a line when given the slope(m) and y-intercept(b), begin by first plotting the y-intercept(b).

Then use the slope(m) to find the next point. Since the slope(m) is positive, count up 4 units. Since the denominator of a whole number is always 1, count right 1 unit. Continue the pattern and graph the line.

Example C

$$y = \frac{1}{2}x + 3$$



To graph a line when given an equation in slope intercept form, first identify the slope(m) and y-intercept(b).

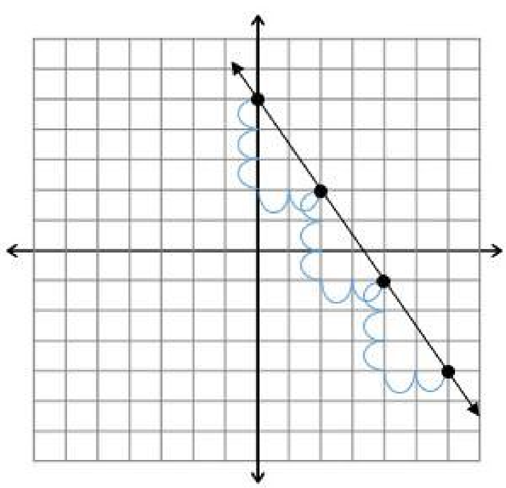
$y = \frac{1}{2}x + 3$

slope(m) = $\frac{1}{2}$
y-intercept(b) = 3

Begin by first plotting the y-intercept(b). Then use the slope(m) to find the next point. Since the slope is positive, count up 1 unit.
Since the denominator is 2, count right 2 units.
Continue the pattern and graph the line.

Example D

$$3x + 2y = 10$$



To graph a line given an equation in standard form, first solve for y and identify the slope(m) and y-intercept(b).

$$\begin{array}{l} 3x + 2y = 10 \\ \underline{-3x \qquad\quad -3x} \\ \frac{2y}{2} = \frac{-3x + 10}{2} \\ y = -\frac{3}{2}x + 5 \end{array}$$

y is being multiplied by 2.
Then 3x is being added.
Work backward.
Subtract 3x from both sides.

Divide both sides by 2.

slope (m) = $-\frac{3}{2}$
y-intercept(b) = 5

Now that you have identified the slope(m) and y-intercept(b), begin by first plotting the y-intercept(b). Then use the slope(m) to find the next point as shown in the examples above. Continue the pattern and graph the line.

Independent Practice

Plot the point, count the slope and then graph the line.

1. (-2, 3) $m = -3$
2. (-3, 3) $m = \frac{2}{5}$
3. (0, -2) m = undefined
4. (0, -3) $m = 0$
5. (-4, 2) $m = -\frac{1}{5}$
6. (-1, -4) $m = 4$
7. (0, 0) $m = \frac{3}{2}$
8. (3, 0) $m = \frac{1}{2}$
9. (0, 0) $m = 1$
10. (-5, 0) $m = \frac{1}{4}$

Graph the line when given the slope (m) and y-intercept (b).

11. $m = 2$ $b = 0$
12. $m = \frac{1}{3}$ $b = 3$
13. $m = \frac{3}{4}$ $b = -4$
14. $m = 0$ $b = 3$
15. $m = \frac{5}{2}$ $b = -2$
16. $m = -3$ $b = 4$
17. $m = -3$ $b = -1$
18. $m = -1$ $b = -1$
19. $m = -\frac{2}{3}$ $b = 1$
20. $m = 0$ $b = -4$

Graph the line given the equation in slope-intercept form y=mx+b. Identify the slope (m) and y-intercept (b).

21. $y = 3x + 1$
22. $y = \frac{-1}{2}x + 4$
23. $y = -3x – 1$
24. $y = \frac{2}{3}x$
25. $y = \frac{1}{3}x + 4$
26. $y = 5$
27. $y = \frac{4}{3}x + 2$
28. $y = x + 4$
29. $y = -\frac{2}{5}x – 1$
30. $y = x$

Graph the equation given in standard form ($Ax + By = C$).

31. $2x + 2y = 10$

32. $x + y = 1$

33. $x + 3y = 12$

34. $2x + y = -3$

35. $2x + 3y = 6$

36. $3x + 4y = 12$

37. $3x - y = 1$

38. $6x - 5y = 20$

39. $x - 4y = 8$

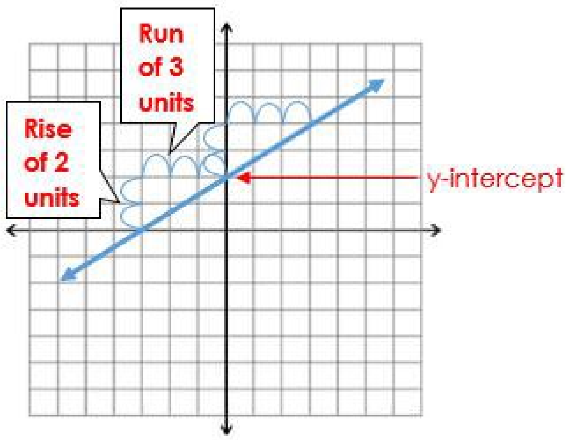
40. $2x - 2y = -6$

- Graph
- Linear
- Slope-intercept form
- Y-intercept

Writing Linear Functions

You will be able to write linear equations in various forms given different constraints.

Example A



To write an equation in slope intercept form of a given line you must find the slope(m) and y-intercept(b).

To find the y-intercept, identify where the line and the y-axis intersect each other. The y-intercept(b) is 2.

To find the slope, find two points.
Calculate the $\dfrac{\textbf{change in y (RISE)}}{\textbf{change in x (RUN)}}$

Begin at the left most point. Count vertically to find the change in y-values (rise). Then count horizontally to the second point to find the change in x-values (run).
The slope(m) is $\frac{2}{3}$.

Now that you have identified the slope(m) and y-intercept(b), you can write the equation by substituting the values.

Slope Intercept Form $y = mx + b$

$$y = \frac{2}{3}x + 2$$

Example B

Slope of 5 and passes through the point (2, 4)

To write an equation of a line in point-slope form when given a point and the slope, use the point-slope formula.

$$y - y_1 = m(x - x_1)$$

The slope(m) is 5 The point is $(\overset{x_1}{2}, \overset{y_1}{4})$

Substitute the values into the formula.

Point Slope Form $\rightarrow y - 4 = 5(x - 2)$

To write the equation in slope intercept form, solve the equation for y.

$$\begin{array}{ll} y - 4 = 5(x - 2) & \text{Distribute the 5} \\ y - 4 = 5x - 10 & \text{Add 4 to both sides.} \\ \quad +4 \qquad +4 & \\ \hline y = 5x - 6 \leftarrow \text{Slope Intercept Form} & \end{array}$$

Example C

A line containing the points (3, -7) and (0, -1).

To write an equation of a line when given 2 points, first use the slope of a line formula to find the slope(m).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Label the ordered pairs. $\overset{x_1\ y_1}{(3, -7)}$ $\overset{x_2\ y_2}{(0, -1)}$

Substitute the values into the formula and simplify.

$$m = \frac{-1 - -7}{0 - 3} = \frac{6}{-3} = -2$$

The slope(m) is -2.

Now that you have identified the slope(m), you can use the slope and one of the points and substitute the values into the point-slope formula.

$$\mathbf{y - y_1 = m(x - x_1)}$$

The slope(m) is -2 A point is $\overset{x_1\ y_1}{(3, -7)}$

Substitute the values into the formula.

$$y - -7 = -2(x - 3)$$

Point Slope Form $\rightarrow y + 7 = -2(x - 3)$

To write the equation in slope intercept form, solve the equation for y.

$y + 7 = -2(x - 3)$ Distribute the -2

$y + 7 = -2x + 6$ Subtract 7 from both sides.

$\underline{\ \ -7 \qquad -7\ \ }$

$y = -2x - 1 \leftarrow$ Slope Intercept Form

Example D

$y = 9x - 2$
$y = 9x + 1$

To determine if 2 lines are parallel or perpendicular, you must first identify their slopes(m).

Lines that are parallel have the same slopes(m).

$y = 9x - 2$
$y = 9x + 1$

The slope(m) in both equations is 9, which means they are parallel.

$y = \frac{4}{5}x + 3$
$y = -\frac{5}{4}x + 4$

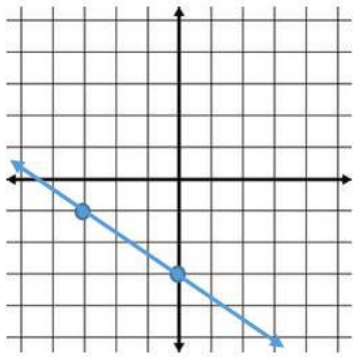
Lines that are perpendicular have opposite reciprocal slopes.

$y = \frac{4}{5}x + 3$
$y = -\frac{5}{4}x + 4$

One slope is positive and one slope is negative, which means they are opposites. The slopes are "flipped", which means they are reciprocals. The slopes are opposite reciprocals, which means they are perpendicular.

Write the equation of the following lines in slope-intercept form.

1.



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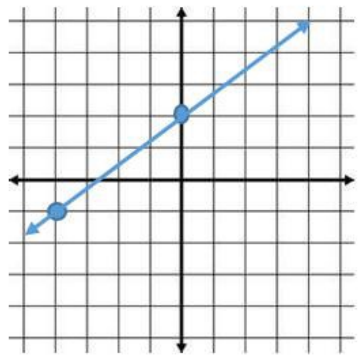
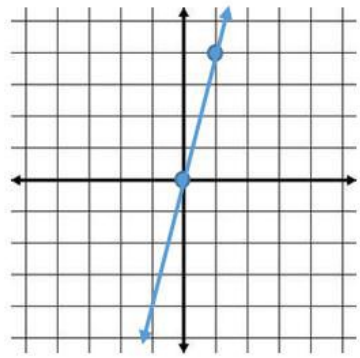


3.



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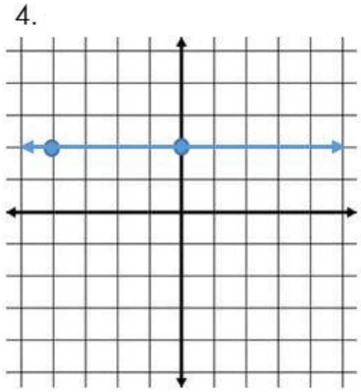

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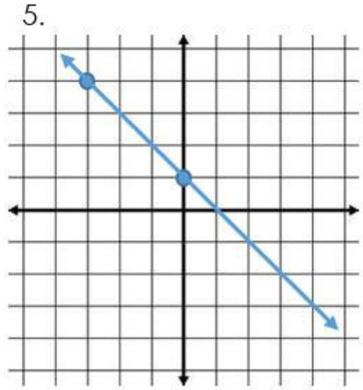

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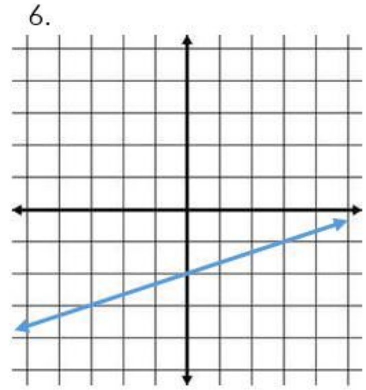
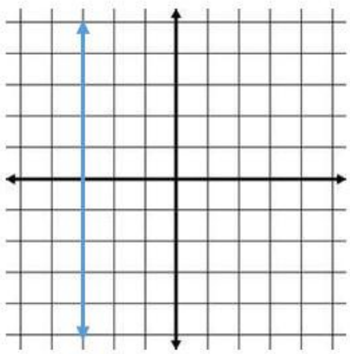


TABLE 1.8: (continued)

7.



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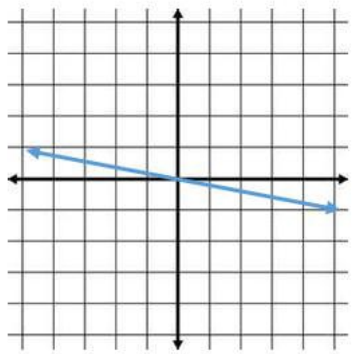

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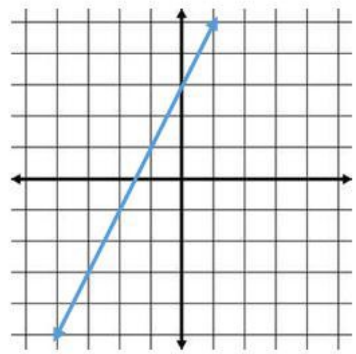
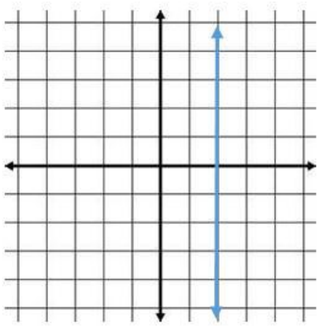


10.



Point-Slope

Write the equation of the line in point-slope form first and then in slope-intecept form.

11. Slope of 1 and passes through the point (-2, 4).
12. Slope of $\frac{1}{3}$ and passes through the point (0, 0).
13. Slope of - $\frac{1}{3}$ and passes through the point (3, 4).
14. Slope of $\frac{1}{2}$ and passes through the point (2, -2).
15. Slope of 5 and a y-intercept of 3.
16. Slope of $\frac{3}{4}$ and passes through (-4, 1)
17. Slope of -$\frac{1}{10}$ and passes through the point (5, -1)
18. Slope of -1 and x-intercept of -1.
19. The line has a slope of 7 and a y-intercept of -2.
20. The line has a slope of -5 and a y-intercept of 6.
21. The line has a slope of -$\frac{1}{4}$ and contains the point (4, -1).

22. The line has a slope of 5 and f(0)= -3
23. m=5, f(0) = -3.
24. m=-7, f(2) = -1
25. m= $\frac{1}{3}$ f(-1) = $\frac{2}{3}$
26. m=4.2, f(-3) = 7.1

Write the equation of the line given 2 points.

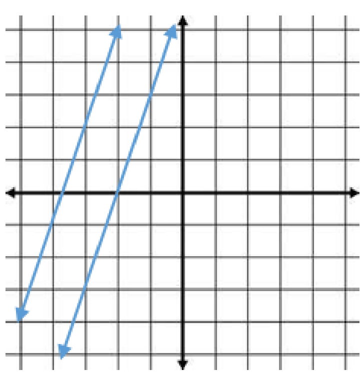
Write the equation of the line in point-slope form $y - y_1 = m(x - x_1)$ first and then in slope intercept form $y - y_1 = mx+b$.

27. The line contains the points (3, 6) and (-3, 0).
28. The line contains the points (-1, 5) and (2, 2).
29. The line goes through the points (-2, 3) and (-1, -2).
30. The line contains the points (10, 12) and (5, 25).
31. The line goes through the points (2, 3) and (2, -3).
32. The line contains the points (3, 5) and (-3, 3).
33. The line contains the points (10, 15) and (12, 20).
34. The line goes through the points (-2, 3) and (-1, -2).
35. The line contains the points (1, 1) and (5, 5).
36. The line goes through the points (2, 3) and (0, 3).
37. A horizontal line passing through (5, 4).
38. A vertical line passing through (-1, 3).
39. x-intercept of 4 and y-intercept of 4.
40. x-intercept of -2 and y-intercept of 5.
41. x-intercept of 3 and y-intercept of 1.

Parallel and Perpendicular LInes

Determine whether the following lines are parallel, perpendicular or neither.

42.



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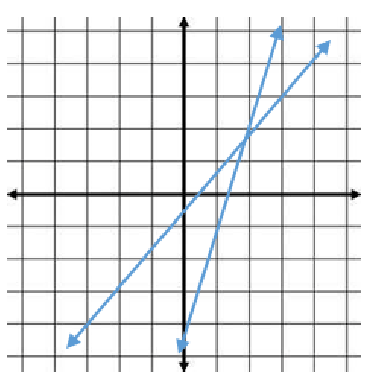

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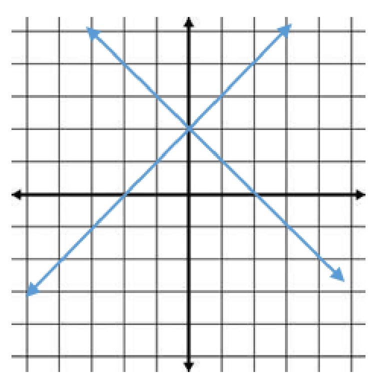

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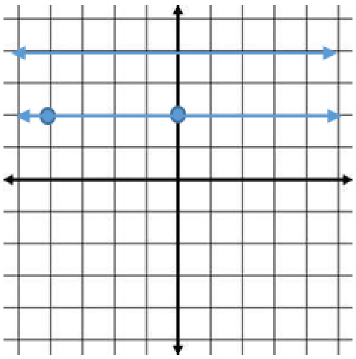

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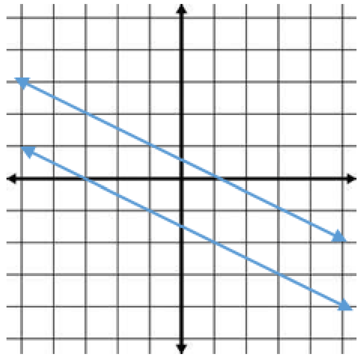
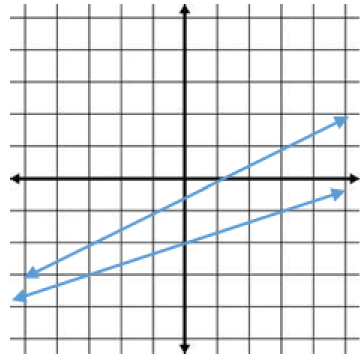


47.



48. $y = 3x + 5$ and $y = -\frac{1}{3}x + 4$

49. $y = 5x - 3$ and $y = -5x - 8$

50. $y = \frac{1}{2}x + 2$ and $y = \frac{x}{2} - 2$

51. $y = x$ and $y = x - 2$

52. $y = \frac{4}{3}x + 5$ and $y = -\frac{4}{3}x + 1$
53. $y = 5$ and $x = 2$
54. $y = -\frac{1}{3}x + 7$ and $y = -3x - 5$
55. $y = -\frac{1}{4}x + 2$ and $y = \frac{1}{4}x + 1$
56. $y = 2x - 4$ and $y = 2x - 7$
57. $y = 4$ and $y = -7$

58. $3y = 12x + 6$ and $10 + y = 4x$
59. $5x + 10y = 20$ and $y = 2x - 7$
60. $y = 3x - 3$ and $y + 7 = -9$
61. $8x + 14 = 2y$ and $7x = 2y + 16$
62. $y = -\frac{1}{5}x$ and $3y = 15x + 3$
63. $18 = 2x - 3y$ and $-5 + y = -\frac{3}{2}x$

- Point-slope formula
- Point-slope form
- Perpendicular lines
- Substituition

Direct Variation.

Identify Direct Variation

The preceding problem is an example of a **direct variation**. We would expect that the strawberries are priced on a "per pound" basis, and that if you buy two-fifths the amount of strawberries, you would pay two-fifths of $12.50 for your strawberries, or $5.00.

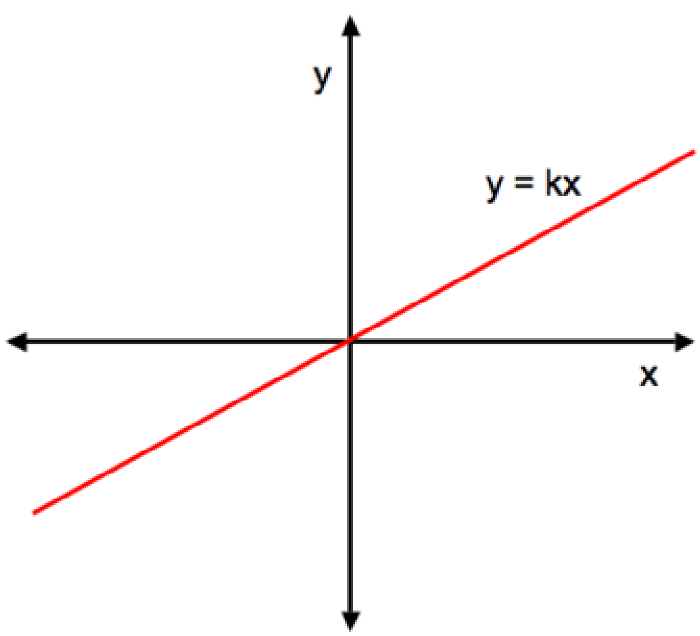
Similarly, if you bought 10 pounds of strawberries (twice the amount) you would pay twice $12.50, and if you did not buy any strawberries you would pay nothing.

If variable y varies directly with variable x, then we write the relationship as

$$y = kx$$

k is called the **constant of variation**

If we were to graph this function, you can see that it would pass through the origin, because $y = 0$ when $x = 0$,whatever the value of k. So we know that a direct variation, when graphed, has a single intercept at (0, 0).



Example A

If y varies directly with x according to the relationship $y = kx$, and $y = 7.5$ when $x = 2.5$, determine the constant of variation, k.

Solution

We can solve for the constant of proportionality using substitution. Substitute $x = 2.5$ and $y = 7.5$ into the equation

$$y = kx$$

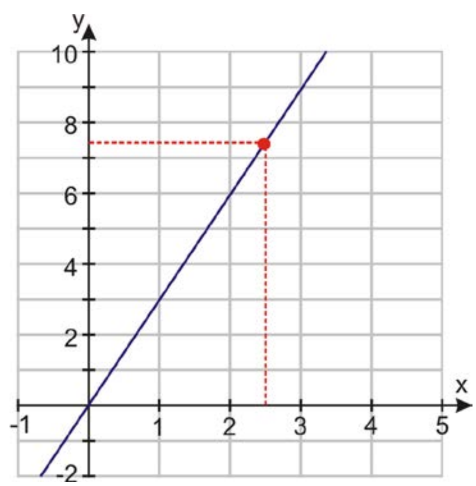
$$7.5 = k(2.5)$$

Divide both sides by 2.5

$$k = \frac{7.5}{2.5} = 3$$

The constant of variation, *k*, is 3.

We can graph the relationship quickly, using the intercept (0, 0) and the point (2.5, 7.5). The graph is shown below. It is a straight line with slope 3.



The graph of a direct variation always passes through the origin, and always has a slope that is equal to the constant of variation, *k*.

Example C

Plot the following direct variations on the same graph.

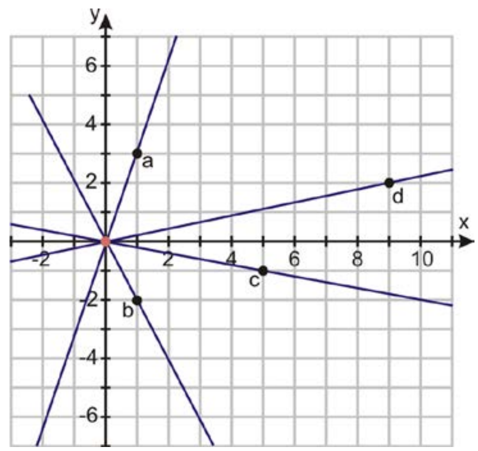
a) $y = 3x$

b) $y = -2x$

c) $y = -0.2x$

d) $y = \frac{2}{9}x$

Solution



All lines pass through the origin (0,0), so this will be the initial point. Apply the slope to determine additional points for each of the equations.

Solve Real-World Problems Using Direct Variation Models

Direct variations are seen everywhere in everyday life. Any time one quantity increases at the same rate another quantity increases (for example, doubling when it doubles and tripling when it triples), we say that they follow a direct variation.

Newton's Second Law

In 1687 Sir Isaac Newton published the famous *Principia Mathematica.* It contained, among other things, his second law of motion. This law is often written as $F = m \cdot a$, where a force of F Newtons applied to a mass of m kilograms results in acceleration of a meters per second^2. Notice that if the mass stays constant, then this formula is basically the same as the direct variation equation, just with different variables—and m is the constant of variation.

Example D

If a 175 Newton force causes a shopping cart to accelerate down the aisle with an acceleration of $2.5\ m/s^2$, *calculate:*

a) *The mass of the shopping cart.*

b) *The force needed to accelerate the same cart at* $6\ m/s^2$.

Solution

a) We can solve for m (the mass) by plugging in our given values for force and acceleration. $F = m \cdot a$ becomes $175 = m(2.5)$, and then we divide both sides by 2.5 to get $70 = m$.

So **the mass of the shopping cart is 70 kg.**

b) Once we have solved for the mass, we simply substitute that value, plus our required acceleration, back into the formula $F = m \cdot a$ and solve for F. We get $F = 70 \times 6 = 420$.

So **the force needed to accelerate the cart at** $6\ m/s^2$ **is 420 Newtons.**

Vocabulary

- If a variable y varies *directly* with variable x, then we write the relationship as $y = kx$, where k is a constant called the **constant of variation.**

Guided Practice

The volume of water in a fish-tank, V, varies directly with depth, d. If there are 15 gallons in the tank when the depth is 8 inches, calculate how much water is in the tank when the depth is 20 inches.

Solution

Since the volume, V, depends on depth, d, we'll use an equation of the form $y = kx$, but in place of y we'll use V and in place of x we'll use d:

$$V = kd$$

We know that when the depth is 8 inches the volume is 15 gallons, so to solve for k, we plug in 15 for V and 8 for d

$$15 = k(8)$$

Divide both sides by 8

$$k = \frac{15}{8} = 1.875$$

Now to find the volume of water at the final depth, we use $V = kd$ again, but this time we can plug in our new d and the value we found for k:

$$V = 1.875(20)$$
$$V = 37.5$$

At a depth of 20 inches, the volume of water in the tank is 37.5 gallons.

Independent Practice.

Explore More

For 1-4, plot the following direct variations on the same graph.

1. $y = \frac{4}{3}x$
2. $y = -\frac{2}{3}x$
3. $y = -\frac{1}{6}x$
4. $y = 1.75x$
5. Dasan's mom takes him to the video arcade for his birthday.

For each equation tell whether *y* varies directly as x.

1. $y = \frac{1}{3}x - 10$

2. $y = 2x$

From the table below tell whether *y* varies directly as x. If so, name the constant of variation and the equation that shows the relationship.

3.

x	.5	8	12	16
y	2	4	6	8

4.

x	2	3	4	5
y	4	3	8	4

Solve. Write the answer to the nearest hundredth.

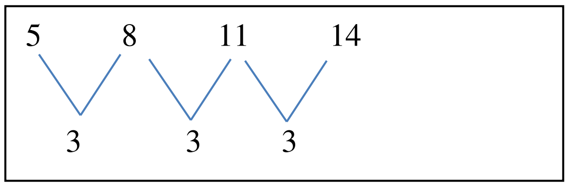
5. *y* varies directly with x. *y* = 6 when x = 1. Find *y* when x = 7.
6. *y* varies directly with x. *y* = 4 when x = 2. Find *y* when x = 6.
7. A is proportional to B. *A* = 6 when B = 2. Find A when B = 1.
8. P is proportional to Q. P = 4 when Q is 1. Find Q when P = 6.
9. S is proportional to R. R = 2.5 when S is 1. Find R when S = 10.
10. D is proportional to C. D = 18 when C = 2. Find D when C = 6.
11. x is proportional to w. x = 2 when w = 0.4. Find x when w = 1.

Arithmetic Sequences as Linear Functions

Some of the patterns we study in secondary math are arithmetic sequences. You can relate the pattern to linear functions. A ***sequence*** is a set of numbers. Each number in the sequence is called a ***term***. Each successive term of an arithmetic sequence is separated by a ***common difference***. For example, if you started with the first term of 5 then added 3, you get 8 for the second term. Then add 8 + 3 to get the next term, and so on. The arithmetic sequence 5, 8, 11, 14… is generated, and the common difference is 3.

In these problems we notice that the first difference is constant, and in this case is equal to 3. (***First difference*** refers to the differences we find the first time we subtract previous terms of the sequence, $(a_n - a_{n-1})$. For linear functions we do not go past the first difference, but to be accurate we will always call it the first difference or common difference.

Note: 3 is a rate of change.



Look at it another way. We could also generate the same sequence by using the expression, 3*x* + 2, and substituting 1, 2, 3, … for *x*.

Now we get to the interesting part. If we did not know the linear expression that generated this sequence, could we find it? Since the first differences between the terms remain constant, this is a linear or arithmetic sequence that could generally be written as $y = ax + b$. Let's substitute 1, 2, 3, … for *x* to generate the general-case sequence.

$$a_1 = 3(1) + 2 = 5 \qquad a_2 = 3(2) + 2 = 8 \qquad a_3 = 3(3) + 2 = 11$$

Note: The first difference is constant (does not change) and is equal to the coefficient of the variable. In other words, it represents the rate of change.

Aha! Does it make sense that the rate of change is the coefficient of *x* in the linear equation *ax + b?* What does the *a* represent in a linear equation?

If *a* = 3, that means that the first term, *3(1)* + *b* = 5. Solving for *b,* 3 + b = 5, so b = 2.

Solution: The sequence was generated by the expression 3*x* + *2.*

Do you see the pattern? Can you describe it and write it?

Here is the pattern (as described by a student): Find the common difference – that is the rate of change (*a*). When you plug in any of the other terms (*x*) to *ax*, for example the first term, the number you get must be adjusted by adding or subtracting to make it work for the value you need.

Arithmetic Sequences as Linear Functions

Sequences Generated by Linear Expressions

Problems 1-4: Write the linear expression that generates the given sequence.

1. $0, 11, 22, 33...$

Hint: Common difference is 11. $11x$ for the first term would be 11(1) or 11. The first term of the sequence should be 0, so we need to subtract 11 to make it work. $11x$-11 would be the formula for finding the n^{th} term. (Try it for the second term: 11(2)=22; 22-11=11. Yes. It works.)

2. $-10, -7, -4, -1, 2, ...$

3. $14, 7, 0, -7, ...$

4. 3, 4, 5, 6, . . .

Problems 5-6: Write the sequence that is generated by the linear expression.

5. $4x+7$

6. $-3x-10$

This instruction may be helpful in finding the function given a linear sequence. For example, in problem #2, the table might look like this:

1	-10	
		3
2	-7	
		3
3	-4	
		3
4	-1	
		3
5	2	

$y = ax + b$

$y = 3x + b$

Then substitute any ordered pair to find b.

$y = 3x - 13$

Solutions:
1. $11x$ - 11
2. $3x$ -13
3. $-7x + 21$
4. $x + 2$
5. 11, 15, 19, 23, . . .

Summarize:

Arithmetic Sequences as Linear Functions

F.BF.2 Build a function that models a relationship between two quantities. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Skill: Use algebraic expressions to identify and describe the *n*th term of a sequence.

Sequence: an ordered list of numbers

Finding the *n*th Term of a Sequence: look for a pattern within the sequence and write the pattern in terms of *n*, which is the term number

Ex: Use the sequence 3, 6, 9, 12, …. Write the next three terms and write an expression for the *n*th term.

Next three terms: We are adding 3 to each term to obtain the next term, so they are $\boxed{15, 18, 21}$.

*n*th term: When $n = 1$, the term is 3; when $n = 2$, the term is 6; when $n = 3$, the term is 9;…

To obtain each term, we multiply the term number by 3. So the expression for the *n*th term is $\boxed{3n}$.

Ex: Use the sequence $12, 8, 4, 0, -4, ...$ Write the next three terms and write an expression for the *n*th term.

Next three terms: We are subtracting 4 to each term to obtain the next term, so they are $\boxed{-8, -12, -16}$.

*n*th term: When $n = 1$, the term is 12; when $n = 2$, the term is 8; when $n = 3$, the term is 4;…

To obtain each term, we multiply the term number by −4 and add 16. So the expression for the *n*th term is $\boxed{-4n + 16}$.

Challenge Ex: Use the sequence 4, 8, 16, …. Write the next three terms and write an expression for the *n*th term.

Next three terms: We are multiplying each term by 2 to obtain the next term, so they are $\boxed{32, 64, 128}$.

*n*th term: When $n = 1$, the term is 4; when $n = 2$, the term is 8; when $n = 3$, the term is 16;…

Arithmetic Sequences as Linear Functions

To obtain each term, we raise 2 to the $n+1$ power. So the expression for the *n*th term is $\boxed{2^{n+1}}$.

<u>You Try</u>: Use the sequence $5, 3, 1, -1, \ldots$ Write the next three terms and write an expression for the *n*th term.

<u>QOD</u>: When a sequence is created by adding or subtracting a number to obtain the next term, where is this number in the expression for the *n*th term?

Sample Questions

1. A company is tracking the number of complaints received on its website. During the first 4 months, they record the following numbers of complaints: 20, 25, 30, and 35. Which is a possible explicit rule for the number of complaints they will receive in the *n*th month?
   a. $a_n = 20n + 5$ b. $a_n = 15 + 5n$

2. Find the 16th term in the following arithmetic sequence. –6, –13, –20, –27, –34, ...
   a. –105 c. –126
   b. –118 d. –111

3. Find the first 5 terms of the sequence with $a_1 = 6$ and $a_n = 2a_{n-1} - 1$ for $n \geq 2$.
   a. 1, 2, 3, 4, 5 c. 6, 12, 24, 48, 96
   b. 6, 7, 8, 9, 10 d. 6, 11, 21, 41, 81

4. Write a rule for the *n*th term of the arithmetic sequence. –10, –4, 2, 8, . . .
   a. $a_n = -16(6)^j$ c. $a_n = -16 + 6$
   b. $a_n = 6n - 16$ d. $a_n = 6n - 18$

5. Find a function that describes the arithmetic sequence 16, 17, 18, 19, ... Use *y* to identify each term in the sequence and *n* to identify each term's position.
   a. $y = 16n$ c. $y = n + 15$
   b. $y = 15n + 1$ d. $y = 17n - 1$

6. What is the first term of an arithmetic sequence with a common difference of 5 and a sixth term of 40?

7. What is the first term of an arithmetic sequence with a common difference of –7 and a seventh term of 40?

8. Write a rule for the *n*th term of the arithmetic sequence.16, 19, 22, . . .

<u>Sample CCSD Common Exam Practice Question(s):</u>

Use the table below to write an equation that represents the value of *y* in terms of *x*.

x	0	1	2	3	4	…
y	3	5	7	9	11	…

A. $y = x + 2$

B. $y = x + 3$

C. $y = 2x + 3$

D. $y = 3x + 2$

Arithmetic Sequences as Linear Functions

Sample Nevada High School Proficiency Exam Questions (taken from 2009 released version H):
These are ONLY challenge exercises for Algebra 1. Look for patterns.

1. The first four terms of a sequence are shown below.

The pattern is $\frac{1}{2x^2}$

$$\frac{1}{2}, \frac{1}{8}, \frac{1}{18}, \frac{1}{32}$$

The sequence continues. What is the seventh term of the sequence?

A $\frac{1}{256}$

B $\frac{1}{98}$

C $\frac{1}{64}$

D $\frac{1}{60}$

2. The first five terms of a sequence are shown below. (Look for patterns of the first differences.)

4 10 28 82 244

The sequence continues. What is the sixth term of the sequence?

A 368
B 486
C 730
D 732

3. The table represents a sequence.

1	**2**	**3**	**4**	**…**	***n***
9	**11**	**13**	**15**	**…**	**?**

Write an equation for the nth term of the sequence.

What is the value of the 12th term of the sequence?

What is the value of the nth term of the sequence?

4. Given the sequence, -2, -1.5, -1, -.5, 0, .5,..., n. What is the value of the 92rd term of the sequence?

Arithmetic Sequences as Linear Functions

We can find the n^{th} term of any sequence. The n^{th} term means that we can find any term in our sequence without writing out every number. The first step is to think of our sequence as a function and create an input-output table, with the input being the term number.
Example:

Term Number

Term Number (Input)	**1**	**2**	**3**	**4**	**5**
Sequence Value (Output)					

Linear Relationships: Proportional vs. Non-Proportional

Verbal Examples

Proportional: Mr. Mangham started the year with \$0. Each week he earned \$25.

Non-Proportional: Mr. Mangham started the year with \$75. Each week he earned \$25.

How to tell the difference:
A proportional situation always starts at zero (in this case \$0 at the first of the year). A non-proportional situation does not start at zero (in this case \$75 at the first of the year).

Table Examples

Proportional:

Weeks	0	1	2	3	4
Money (\$)	0	25	50	75	100
$\frac{\text{Money}}{\text{Weeks}}$	---	25	25	25	25

Non-Proportional:

Weeks	0	1	2	3	4
Money (\$)	75	100	125	150	175
$\frac{\text{Money}}{\text{Weeks}}$	---	100	62.5	50	43.75

How to tell the difference:
A proportional table has a constant of proportionality in that y divided by x always equals the same value. A non-proportional table will have different values when y is divided by x.

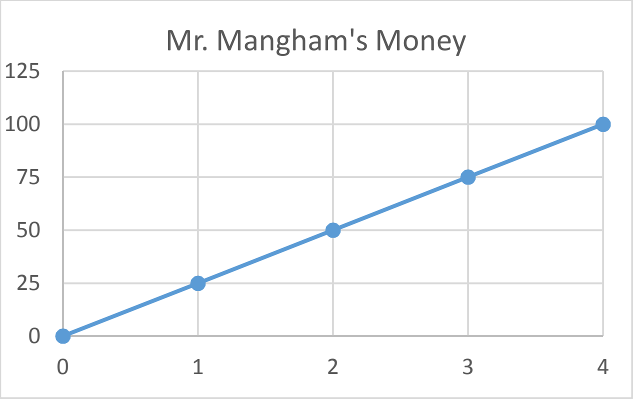
Equation Examples

Proportional: $y = 25x$ *Non-proportional:* $y = 25x + 75$

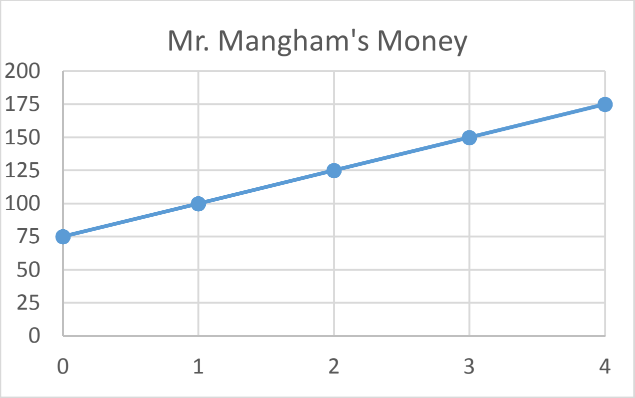
How to tell the difference:
A proportional equation is always in the form $y = kx$, where k is the unit rate or constant of proportionality. A non-proportional equation is always in the form $y = mx + b$, where m is the constant rate of change or slope. The key difference is the added b on the end.

Graph Examples

Proportional:



Non-Proportional:



How to tell the difference:
A proportional graph is a straight line that always goes through the origin.
A non-proportional graph is a straight line that does not go through the origin.